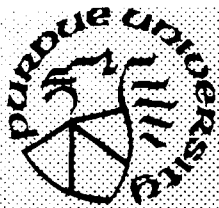




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# Functional Characterization of Fault Tolerant Integration in Distributed Sensor Networks<sup>†</sup>

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# FUNCTIONAL CHARACTERIZATION OF FAULT TOLERANT INTEGRATION IN DISTRIBUTED SENSOR NETWORKS<sup>†</sup>

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## Abstract

Fault-tolerance is an important issue in network design because sensor networks must function in a dynamic, uncertain world. In this paper, we propose a functional characterization of the fault-tolerant integration of abstract interval estimates. This model provides a test bed for a general framework which we hope to develop to address the general problem of fault-tolerant integration of abstract sensor estimates. We further propose a scheme for narrowing the width of the sensor output in a specific failure model and give it a functional representation.

The main distinguishing feature of our model over the original Marzullo's model is in reducing the width of the output interval estimate significantly in most cases where the number of sensors involved is large.

**Keywords and phrases:** Distributed Sensor Networks, Function Representations, Fault-Tolerant Model, Abstract sensor, Interval estimate.

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## 1.0 INTRODUCTION

In recent years, the increasing sophistication of surveillance systems and tracking mechanisms has generated a great deal of interest in the development of new computational structures and strategies for detecting and tracking multiple targets, using data from many sensors.

The design of spatially distributed target-detection-and-tracking systems involves the integration of solutions obtained by solving subproblems in data-association, hypothesis-testing, data-fusion, etc. [13]. This must include the cooperative solution of problems by a decentralized and loosely coupled collection of processors, each of which integrates information received from a cluster of spatially distributed sensors into a manageable and reliable output for further integration at a higher level. Integration of information at the sensor level requires techniques to be developed to abstractly represent and integrate sensor information. Further these techniques have to be robust in the sense that even if some of the sensors are faulty, the integrated output should still be reliable. For details on multi sensor integration and fusion in intelligent systems, see [ 5,6,7,8,9,10,11,14].

The aim of this paper is to present a fault-tolerant computational model for sensor integration in *Distributed Sensor Networks*.

A *Distributed Sensor Network* (DSN) consists of spatially distributed sensors which detect and measure a certain phenomenon via its changing parameters. These readings are sent at regular intervals of time to processing units which *integrate* the readings from clusters of sensors and give outputs whose nature is much the same as the inputs of the sensors. Outputs from processors representing clusters of sensors are later integrated to get a complete picture of the spatially distributed phenomenon. However, before integration is performed at the processor level, it is necessary to have reliable estimates at each processor. Each sensor in a cluster measures the same set of parameters. It is possible that some of these sensors are faulty. Hence it is desirable to make use of the redundancy of the readings in the cluster to

obtain a correct estimate of the parameter being read. In short, a *fault-tolerant* technique of sensor integration to obtain the correct estimate is sought.

## **1.1 Scope of This Paper**

This paper has two objectives: The first objective is to propose a functional characterization of fault-tolerant integration of abstract interval estimates considered by Marzullo [4]. The second objective is to propose a modified computational scheme of integration carried out by Marzullo [4] in the case when the number of sensors is large, wherein it is possible to improve the accuracy of the integrated output.

The main distinguishing feature of our model over the original Marzullo's model is in reducing the width of the output interval estimate significantly in most cases where the number of sensors involved is large.

Elsewhere we intend to generalize Marzullo's approach to the cases when the sensor outputs are subsets of an abstract parameter space. The functional characterization of the fault-tolerant integration of abstract interval estimates described in this paper hints at an abstract framework. We hope to develop for addressing the general problem of fault-tolerant integration of sensor outputs.

## **1.2 Organization of the Paper**

In section 2, we describe Marzullo's work on sensor integration and other related work. Our abstract model functional characterization is detailed in section 3 and is an extension of the model proposed by Marzullo. In section 4, we motivate the need for a new failure model and present the information integration algorithm with a specific example. Finally, we close the paper with concluding remarks and future directions this reach would take.

## **2.0 RELATED WORK**

Marzullo [4] considers the case of a processor receiving input from several sensors whose outputs are connected intervals. He gives a fault-tolerant integration algorithm which takes as input the intervals representing the sensors and gives as output of the processor a connected interval representing the sensor values. More precisely: Let there be  $n$  sensors, each of which yields an interval as its output. these sensors measure a certain

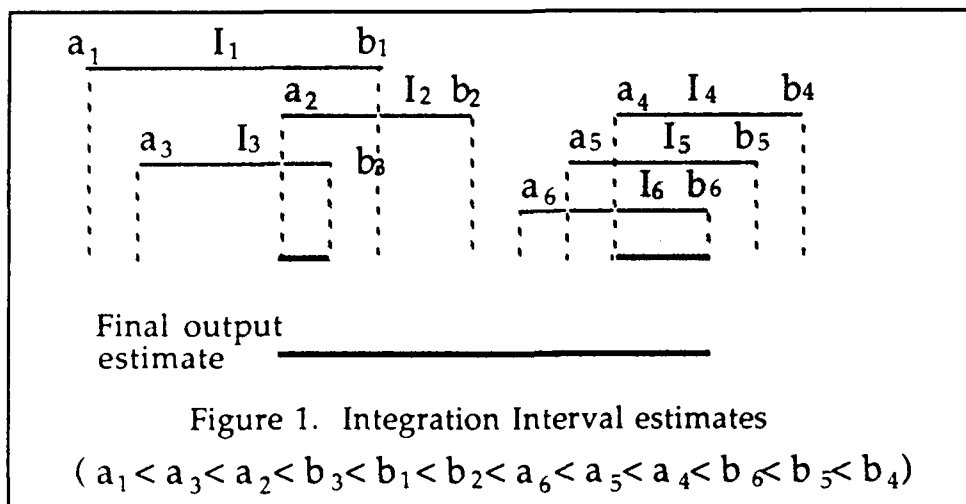
physical value and their intervals contain the physical value unless they happen to be faulty sensors.

Thus, a correct sensor is one which contains the actual physical value in its interval. Any two correct sensors must overlap since they both contain the physical value being measured.

Marzullo considers the case when almost  $f$  sensors are faulty and gives an algorithm which yields a connected interval as the output of the processor, containing the physical value.

If at most  $f$  of the  $n$  sensors are faulty, then it follows that at least  $n-f$  sensors are correct. Marzullo considers all possible nonempty  $(n-f)$ -intersections of the  $n$ -sensors. A sensor which does not belong to any of the  $(n-f)$ -cliques is faulty since a correct sensor overlaps with at least  $(n-f-1)$  other correct sensors. One and only one of the  $(n-f)$ -intersections contains the physical value. Since it is not possible to decide which intersection has the physical value (which is as yet unknown to us) and since the processor output is required to be a connected interval, the smallest connected interval containing all the  $(n-f)$ -intersections is taken to be the output of the processor. It is easy to see that it contains the actual physical value. The wider this interval is, the lesser the accuracy of the processor output. Marzullo proves the existence of bounds for the width of this interval in terms of  $f$ .

The example described below provide a description of integration process.



In the above figure, we have the intervals  $I_j = [a_j, b_j]$   $1 \leq j \leq 6$ . Overlapping one another according to the strict chain of inequalities given above. Here  $f = 3$  and  $n = 6$ . So, taking all possible  $(n - f)$  intersections gives us the intervals  $[a_2, b_3]$  and  $[a_4, b_6]$ . Then enclosing these intervals in the smallest possible connected interval, we have the integrated output interval  $I_p$  given by  $I_p = [a_2, b_6]$ .

In the statistical literature, the popular methods for the combining the point estimates of several (possibly faulty) sensors into a single point estimate come under the designation of robust estimation. This family includes the median, Huber function based methods, least median square methods, etc. [1,2]. Some of these methods have been applied to the sensor fusion problem in [3].

However, in methods like a median, there is no easy way of including the additional information such as "at most  $f$  out of  $n$  sensors are faulty". Moreover, all of these methods are geared to the generation of point estimates whereas our paper concentrates on the interval estimates.

The main thrust in our paper is in the derivation of computational schemes for narrowing the width of the processor output in a specific failure model and give it a functional representation.

## **2.1 Interval Representation of Sensor Readings**

A sensor reads a physical variable and gives a number as its output. However a sensor is prone to inaccuracies and there may be some uncertainty in the value of its output. The simplest modeling of this is achieved by looking upon sensor outputs as connected intervals on the real line rather than as points. The actual value representing the physical variable being measured is taken to be contained in the interval associated with the sensor if the sensor is not faulty. No assumptions are made about the width of these intervals or their position on the real line. Thus each sensor value is represented by an interval estimate. We make this notion precise in the

following definitions that are useful in characterizing one model of sensor integration.

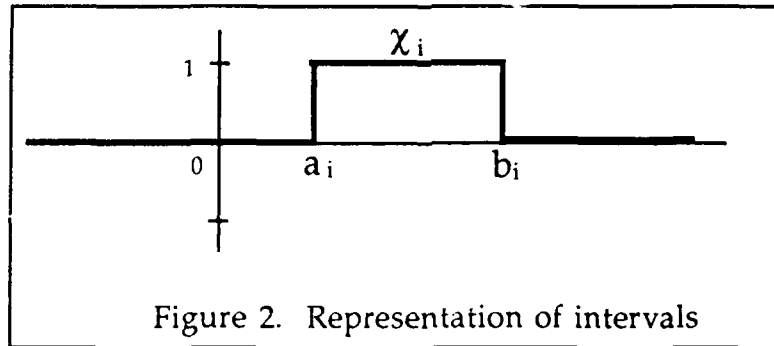
**Definition 1:** An *abstract sensor* is a sensor which reads a physical parameter and gives out an abstract interval estimate  $I_s$ , which is a bounded and connected subset of the real line  $\mathbf{R}$ .

**Definition 2:** A *correct sensor* is an abstract sensor whose interval estimate contains the actual value of the parameter being measured. If the interval estimate does not contain the actual value of the parameters being measured, it is called a *faulty sensor*.

**Definition 3:** Let sensors  $s_1, \dots, s_n$  feed into a processor  $P$ . Let the abstract interval estimate of  $s_j$  be  $I_j$   $1 \leq j \leq n$ , where  $I_j$  is the closed interval  $[a_j, b_j]$  with end-points  $a_j$  and  $b_j$ . Define the characteristic function  $\chi_j$  of the  $j^{\text{th}}$  sensor  $s_j$   $1 \leq j \leq n$  as follows :

$$\chi_j: \mathbf{R} \rightarrow \{0,1\}$$

$$\chi_j(x) = \begin{cases} 1 & \forall x \in I_j \\ 0 & \forall x \notin I_j \end{cases} \quad \forall 1 \leq j \leq n.$$



The next section addresses the question of how the abstract sensors or abstract estimates are combined to yield new abstract estimates.

### 3.0 THE PROBLEM OF FAULT-TOLERANT SENSOR INTEGRATION

Fault tolerance is a crucial requirement to be satisfied by a distributed sensor network for it to be reliable in a situation where one or more of its



sensors fails and yield faulty readings. Fault tolerance in a distributed sensor network thus implies that the values of the parameters measured by the network are reliably reflected in its output even though some of the sensors may be faulty. We propose to introduce fault tolerance into the distributed sensor network by providing a method of integration of sensor values that yields a reliable output which reflects the correct values of the parameters being measured with high fidelity.

Recall that our definition of a faulty sensor is one where abstract sensor estimate (interval estimate) does not count in the actual physical value being measured.

The problem of fault-tolerant sensor integration is the integration of the  $I_j$  ( $1 \leq j \leq n$ ), to obtain an abstract interval estimate  $I_p = [a_p, b_p]$  which is a 'reliable' and 'fairly accurate' estimate of the region in which the physical sensor value lies. This integration should be fault-tolerant in that its reliability should not be severely affected by some of the sensors being faulty. In other words, we seek to obtain a functional relationship between the characteristic function  $\chi_p$  of  $I_p = [a_p, b_p]$  and the  $\chi_j$   $1 \leq j \leq n$ :  $\chi_p(x) = f(\chi_1(x), \dots, \chi_n(x))$  such that  $\chi_p^{-1}(1)$  is a fault-tolerant interval estimate of the physical value being measured.

We now go about obtaining a functional representation of the integrated output estimate under the integration scheme of Marzullo. In order to do this we need to introduce a few relevant operations and functions. The following definition provide such operations for our integration problem:

**Definition 1:** If  $f(x)$  is a real-valued function, define  $\|f\| = \sup\{|f(x)| \mid x \in \mathbf{R}\}$  (norm of  $f$ ). Here  $\sup$  stands for the supremum. That is,  $\|f\|$  is the smallest real number  $\alpha$  such that  $f(x) \leq \alpha \forall x \in \mathbf{R}$ .

**Definition 2:** If  $f(x)$  is a real valued function define  $\text{Supp } f = \{x \mid f(x) \neq 0\}$  (support of  $f$ ).

**Definition 3:** Let  $O(x) = \sum_{j=1}^n \chi_j(x)$  to be the 'overlap function'. For each  $x \in \mathbb{R}$ ,  $O(x)$  gives the number of intervals in which  $x$  lies or the number of intervals overlapping at the point  $x$ .

**Remark 1:** The integer  $\|\chi_i O(x)\|$  gives the maximum number of intervals in which any  $x \in f_i$ .

**Proof:** Indeed,  $O(x)$  is the number of intervals overlapping at the point  $x$ . Multiplying by  $\chi_i(x)$  restricts  $O(x)$  to the interval  $I_i$

$\|\chi_i O\|$  therefore gives the maximum number of intervals overlapping at any  $x \in f_i$ .

Then  $I_i$  belongs to  $\|\chi_i O\|$ -clique. Where, by a  $n$ -clique we mean a group of  $n$  intervals having a common intersection. ■

If  $\chi_i(x)$  and  $\chi_j(x)$  are characteristic functions of intervals  $I_i$  and  $I_j$  then the characteristic function of the interval  $I_i \cap I_j$  denoted  $\chi_{I_i \cap I_j}(x)$  is given by the product  $\chi_i(x) \chi_j(x)$ .

If  $\chi_i(x)$  is the characteristic function of  $I_i$ , then the characteristic function of  $I_i^c$  (the complement set of  $I_i$ ) is given by  $(1 - \chi_i(x))$ .

Thus if  $I_1, \dots, I_n$  are intervals with characteristic functions  $\chi_1, \dots, \chi_n$ , then the characteristic function of their union  $\chi_{\cup I_i}(x)$  is given by

$$\chi_{\cup I_i}(x) = 1 - \prod_{i=1}^n (1 - \chi_i(x)).$$

Marzullo[4] assumes that there are at most  $f$  faulty sensors among  $n$  sensors, and considers the intersections of  $(n-f)$  or more sensors as the regions in which the correct physical value lies. An interval which does not participate in any  $(n-f)$ -intersection is taken to be the estimate of a faulty sensor. The output is the smallest interval which contains all  $(n-f)$  or more intersections.

### 3.1. Computational Characterization

We now obtain a functional characterization of the  $(n - f)$  intersections and this integrated output estimate as described in the previous section. More precisely, we give an explicit expression for this characteristic function of the  $(n - f)$  intersection in terms of the characteristic function of the intervals corresponding to the sensor estimates.

**Remark 2:** If at most  $f$  sensors are faulty, then we need to consider only those  $I_i$ 's for which  $\|\chi_i O\| \geq (n-f)$ . Thus the characteristic function of the set of all points lying in  $(n-f)$  or more intersections of the intervals  $I_j$  ( $1 \leq j \leq n$ ) is given by :

$$S(x) = 1 - \prod_{j=1}^n \left( 1 - \chi_{[n-f, \infty)}(\|\chi_i O\|) \chi_j(x) \right)$$

Where  $\chi_{[n-f, \infty)}$  is the characteristic function of the interval  $[n-f, \infty)$ .

**Proof:** Indeed  $\chi_{[n-f, \infty)}(\|\chi_i O\|) = 1$  iff  $\|\chi_i O\| \geq n-f$

i.e. iff  $I_j$  has at least  $n-f-1$  other intervals intersecting it at some point in it.

So  $\chi_{[n-f, \infty)}(\|\chi_i O\|) \chi_j(x) = \chi_j(x)$  if and only if  $I_j$  is as described above. So that  $S(x)$  is the support of the characteristic function of all those points which belong to  $(n-f)$  or more intervals' intersection. ■

Now the correct physical value belongs to  $\text{Supp}(S(x))$ , i.e; to one of the intervals constituting it. Marzullo proposes the smallest connected interval containing  $\text{Supp}(S(x))$  for the integrated output.

More precisely the output interval estimate  $I_p$  is given by :

$$I_p = [ \text{Min} \{ x \mid S(x) = 1 \}, \text{Max} \{ x \mid S(x) = 1 \} ]$$

The above integration technique does indeed give a connected Interval within which the actual physical values lies. It however includes points which do not belong to the intervals constituting  $\text{Supp}(S(x))$ .

Furthermore, if the intervals constituting  $\text{Supp}(S(x))$  are widely scattered over  $\bigcup_{i=1}^k I_j$ , then there are wide gaps between these intervals which do not contain the physical value and yet are included in the output estimate. This results in the width of the smallest interval containing  $\text{Supp}(S(x))$  being comparable to the smallest interval containing  $\bigcup_{i=1}^k I_j$  (see Fig. 1) and this is of little value from the point of accuracy. We need to evolve a criterion by which we can pick only a few of the intervals constituting  $\text{Supp}(S(x))$  with a fair amount of certainty that they enclose the correct physical value. The rest of our analysis is in this direction.

#### **4.0. A NEW FAILURE MODEL WITH SHARPER OUTPUT INTERVAL ESTIMATES**

We propose a failure model in which it is possible to choose in most cases a subset of  $\text{Supp}(S(x))$  as the region of correct sensor value instead of the whole of  $I_p$  as defined above. A sensor may fail *wildly*, in which case there is no correlation between the actual physical value being measured and the interval estimate of the faulty sensor. On the other hand, a sensor may fail *tamely*, in which case although the faulty sensor's interval estimate does not contain the actual physical value, the interval estimate lies significantly close to the value in a certain sense. For example, mechanical vibrations may induce a tame fault in dials and meters by shifting the needless fluctuations to a region which does not contain the correct value but lies close to it. Since we do not know the actual physical value, we cannot detect the tameness of a fault directly. However tamely faulty sensor estimates tend to overlap with correct sensor estimates because of their proximity to the actual physical value. We consider the case when the number of sensors to be integrated is very large and assume that most of the faulty sensors are tamely faulty. In this case, we observe that correct sensors have a relatively larger number of intervals overlapping with them as compared to undetected faulty sensors participating in the  $(n-f)$ -intersections, since tamely faulty sensors overlap

with correct sensor estimates. Thus the number of sensor estimates overlapping with a given sensor estimate is a good index of its correctness. We make use of this observation to narrow our output interval estimate, namely  $I_p$ .

Let  $\text{Supp}(S(x)) = \bigcup_{i=1}^k L_i$  where  $L_i = [\alpha_i, \beta_i]$  with  $\beta_i < \alpha_{i+1} \forall 1 \leq i \leq k-1$ . We

now perform an evaluation of the  $L_i$ 's in order to attach a weight to each of them and choose those  $L_i$ 's with maximum weight to be the intervals which have a high likelihood of containing the correct physical value. We then again enclose these  $L_i$ 's of maximum weight by the smallest possible interval and take it to be the output estimate.

**Remark 3:** Let  $\chi_{L_i}(x)$  be the characteristic function of  $L_i$ . Then we can define

the *popularity* of the  $j^{\text{th}}$  sensor to be the number  $P_j = \sum_{k=1}^n ||\chi_k \chi_j|| -$

$1 \cdot P_j$  gives the number of sensor intervals overlapping with the  $j^{\text{th}}$  sensor interval.

**Proof:** Indeed  $||\chi_k \chi_j|| = \begin{cases} 1 & \text{if } I_k \cap I_j \neq \emptyset \\ 0 & \text{if } I_k \cap I_j = \emptyset \end{cases}$

Thus  $\sum_{k=1}^n ||\chi_k \chi_j|| - 1$  gives the number of intervals (apart from  $I_j$ ) intersecting with  $I_j$ . ■

#### 4.1 Narrowing of the Output Interval Estimate Width

The  $L_i$ 's are  $(n - f)$  intersections. The reliability  $r_i$  gives a measure of the clustering of sensors around the  $L_i$ . Our assumption that most of the faulty sensors are tamely faulty implies that the  $L_i$  with maximum clustering around it is most likely to contain the correct physical value measured by the sensors. The sum of the popularities of the sensors involved in the formation of  $L_i$  is a good index of the clustering of sensors about  $L_i$ .

Hence, we would like to take the sum of the popularities of all sensors involved in the formation of  $L_i$ , and call it the *reliability*  $r_i$  of  $L_i$ .

Consider the set function  $W(L_i) = \sum_{j=1}^n \|\chi_{L_i} \chi_j\| P_j$ ,  $1 \leq i \leq k$  defined on each  $L_i$ .  $W(L_i)$  gives the sum of the number of intervals overlapping with each sensor estimate in the (n-f)-or-more clique  $L_i$ .

$$\text{i.e., } r_i = W(L_i) \quad \forall 1 \leq i \leq k.$$

Let  $r = \max \{r_i \mid 1 \leq i \leq k\}$ ,  $m = \min \{i \mid r_i = r\}$  and  $M = \max \{i \mid r_i = r\}$ . Consider the interval  $[\alpha_m, \beta_M]$ . We take  $I_p^* = [\alpha_m, \beta_M]$  as the integrated output estimate.

It is clear that  $\beta_M - \alpha_m = |I_p^*| \leq |I_p|$ , where  $|I|$  is the width of the interval  $I$ . Thus in our failure model we have in general a way of narrowing the output estimate  $I_p$  to  $I_p^*$ . However if the number of wildly faulty sensors are as many as the tamely faulty ones, and if they happen to cluster somewhere else on  $I_p$ , then it is possible that  $|I_p^*| = |I_p|$ . Thus the worst case for  $I_p^*$  is  $I_p$ . The chances that wildly faulty sensors mimic the clustering behavior of tamely faulty sensors are remote. Also if the number of sensors is very small, it is possible that  $|I_p^*| = |I_p|$ .

For example, consider the case of three input sensor estimates  $I_1=[2.4,3.2]$ ,  $I_2=[2.9,4]$ ,  $I_3=[3.6,5]$ . In this case  $I_p=[2.9,4]$ . Here  $L_1=[2.9,3.2]$  and  $L_2=[3.6,4]$ , but they both have the same reliability. Hence  $I_p^*=I_p$  here.

## 4.2 The Algorithm

We now present the algorithm as follows:

### Algorithm :

**Input :** Intervals  $I_1, I_2, \dots, I_n, f$ .

**Output:** Integrated output estimate.

**begin**

1. **Take** all  $(n-f)$ -intersections of the intervals to yield Intervals  $L_1, \dots, L_k$ , each of which is an  $(n-f)$ -intersection :  $\{ L_j = [a_j, b_j] \}$ ;
2. **For** each  $i$  ( $1 \leq i \leq k$ )
  - i. **Count** the number of intervals intersecting each of the intervals  $I_j$  ( $1 \leq j \leq n$ ) having nonempty intersection with  $L_i$ ;
  - ii. **Add** these numbers up to obtain a number  $r_i$  {  $r_i$  gives the sum of the number of intervals intersecting with intervals involved in the formation of the  $L_i$ .  $r_i$  is a measure of the reliability of  $L_i$  };
3. **Choose** the maximum of the  $r_i$  ( $1 \leq i \leq k$ ) and call it  $r$ ;
4. **Let**  $m = \min\{ i \mid r_i = r \}$  and  $M = \max\{ i \mid r_i = r \}$ ;
5. **Assign**  $I_p^* = [a_m, b_M]$  to be the integrated output estimate;

**end.**

## 4.3 A Comparison of Performance

In this new model, we find that when the number of sensors is very large, by taking the clustering of the tamely faulty sensors into consideration, we reduce the output intervals width greatly, as compared to Marzullo's [4] output interval estimate.

The Figure 3 below illustrates the superior performance of our model clearly. The numbers near each interval estimate gives the number of

intervals overlapping with it. Here  $n = 13$  and  $f$  (maximum number of fault intervals allowed) = 10. The thick lines in Figure 3 are the intervals  $I_i$  with the numbers on them indicating their reliabilities. We may pick either the interval with higher reliability or define a range for reliabilities and pick intervals which fall in these limits. The thick line of the bottom indicates the output interval estimate for this case in Marzullo's model.

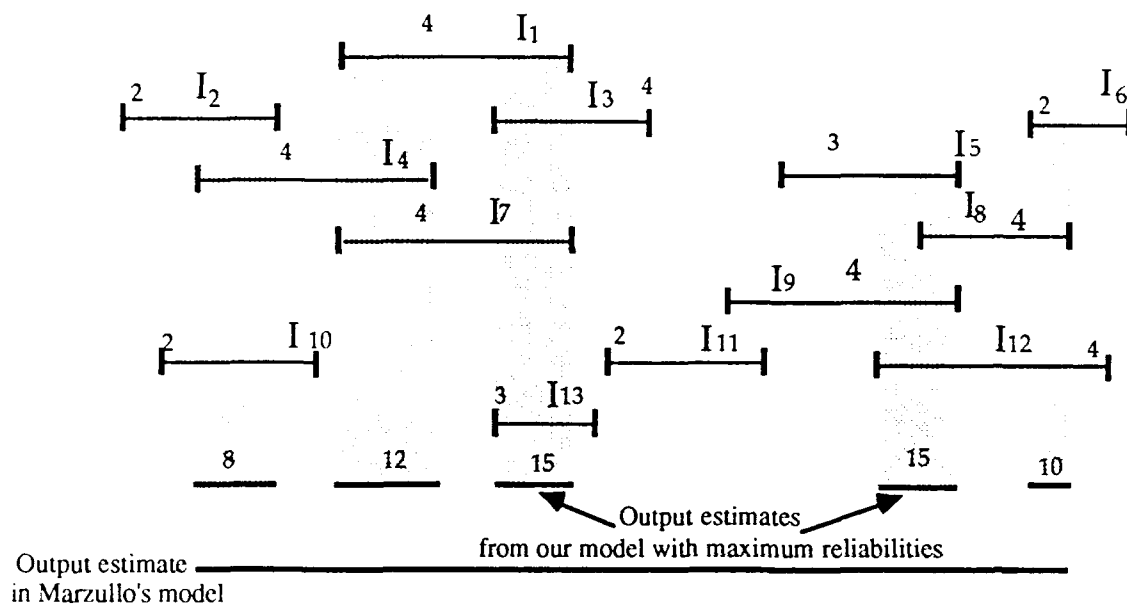


Figure 3. A comparison of performance

**Table 1.** Popularities of Intervals

Interval	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$	$I_8$	$I_9$	$I_{10}$	$I_{11}$	$I_{12}$	$I_{13}$
Popularity	4	2	4	4	3	2	4	4	4	2	2	4	3

The essential gain in the model proposed above is that the  $(n-f)$ -intersections are assigned weights which are in the above mentioned sense reliability estimates of these intersections. We may now impose any convenient rule for choosing these segments according to their reliabilities.



In table 1, we have each interval assigned a numbering which is its popularity.

The  $(n - f)$  or more intersections (i.e. 3 to 4-intersections here) which form the output have reliabilities 8, 12, 15, 15, 10.

Thus, the advantage here is that we have the intersection weighted to help us judiciously choose the output intervals. We may employ any convenient rule depending upon our faith in the tameness of the faults to pick these intervals and enclose them by connected interval. For instance, we may choose only those intervals with maximum reliability (in this case, the intervals with reliability 15) and enclose them by a connected interval. It is clear that the worst possible width for the final output interval estimate is the smallest interval containing all intersections irrespective of their weights.

#### 4.3 Narrowing width of Intervals in the case of non-uniform distributions

So far we have assumed that a sensor does not give any information about the probability distribution of the correct physical value over the interval (abstract sensor estimate) that represents it. In this case, we assume that each value in this interval is equally likely, i.e., we assume a uniform distribution of the physical value over each interval estimate. However, in the event of the sensor giving a unimodal probability distribution of physical value over its interval estimate, say, for example, a normal distribution, then we may use this additional information to narrow the interval widths beforehand by restoring to confidence interval estimates of the physical value. The resulting subset of this estimation is a connected interval whose width is less than the original interval. More precisely, if  $S$  is a sensor with its interval estimate  $I = [a, b]$  and  $p(x)$  is a unimodal probability distribution on  $I$  of the physical value measured by the sensor, and further if  $(1 - \alpha)$  is a high probability, we need to compute members  $L(\alpha), U(\alpha)$  such that

$$P[x \in [L(\alpha), U(\alpha)]] = 1 - \alpha$$

$$= \frac{U(\alpha) - L(\alpha)}{\int_{L(\alpha)}^{U(\alpha)} p(x)dx}$$

where  $[L(\alpha), U(\alpha)]$  is called  $100(1 - \alpha)\%$  confidence interval for the physical value  $x$ , and  $(1 - \alpha)$  is called the level of confidence associated with the interval.

Now,  $[L(\alpha), U(\alpha)] \subseteq [a, b]$  if  $\alpha \neq 0$  with a high probability of containing the physical value (if  $S$  is not faulty) and if is narrower than the original interval  $[a, b]$ . It is obvious that the widths of the  $L_i$  decrease if the width of one or more of the input intervals decrease. Thus the narrowing of the intervals before integration when possible increases the accuracy of the output.

## 5.0 CONCLUDING REMARKS

In order to address the general problem of fault-tolerant sensor integration for a large class of sensors, it is necessary to evolve a broad-based computational framework which can accommodate a wide range of sensors and a variety of fault tolerant integration techniques depending upon the phenomenon being sensed and the method of sensing. We intend to develop a calculus of sensor integration by regarding the sensor estimates as subsets of an abstract parameter space and obtaining functional representations of the characteristics of these estimates. We then intend to obtain rules for combining these functions to get functions describing the characteristics of the output according to the kind of integration that is required to be performed. This paper is a preliminary exercise in concert with this effort. We have recast the fault-tolerant integration of abstract interval estimates *a la* Marzullo [4] in a computational framework, and considered a failure model wherein we could reduce the width of the output interval estimate significantly in most cases where the number of sensors involved is large.

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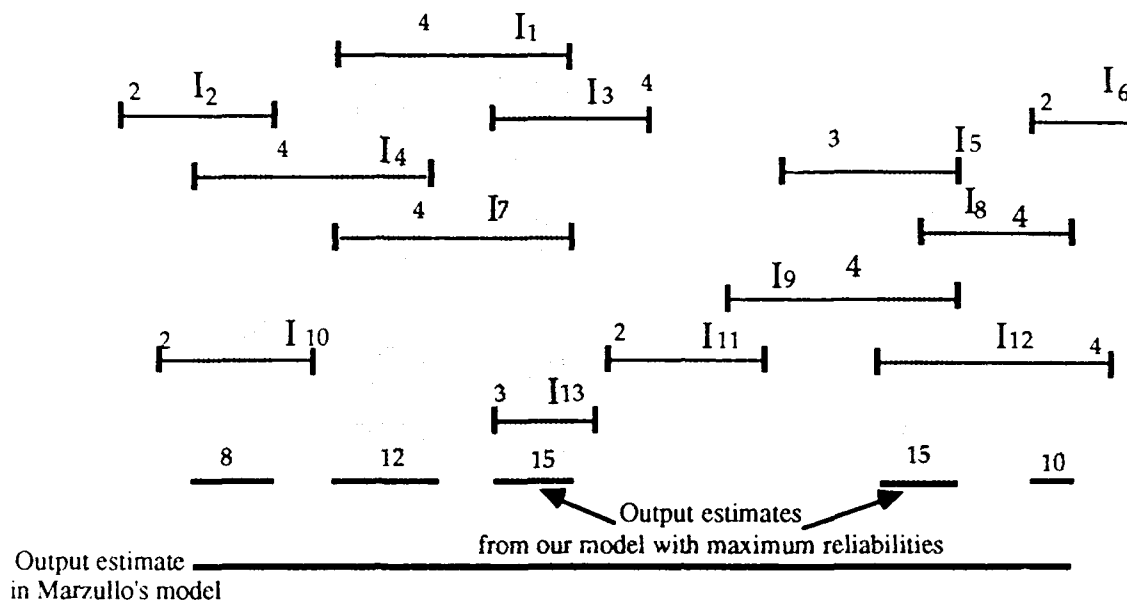


Figure 3. A comparison of performance

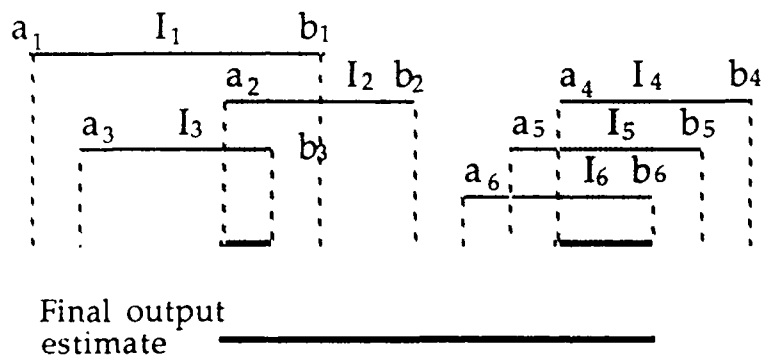


Figure 1. Integration Interval estimates

$$(a_1 < a_3 < a_2 < b_3 < b_1 < b_2 < a_6 < a_5 < a_4 < b_6 < b_5 < b_4)$$

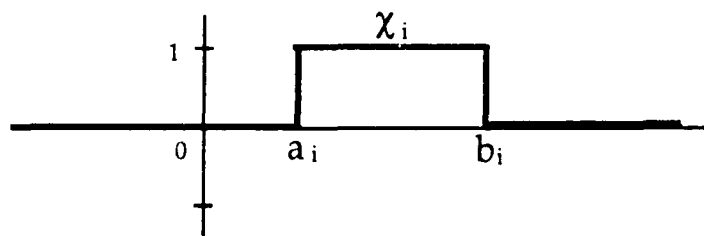


Figure 2. Representation of intervals